

Določeni integral: rešene naloge, formule, skice

1. Izračunaj integrale.

(a) $\int_0^\pi \sin x \, dx$

(b) $\int_0^1 x^4 \, dx$

(c) $\int_1^4 (x+1)x^{-1} \, dx$

(d) $\int_1^4 2\sqrt{x} \, dx$

(e) $\int_1^4 \frac{x+\sqrt{x}+1}{x} \, dx$

$$\int_a^b f(x) \, dx = F(x) \Big|_a^b = F(b) - F(a)$$

ZGLEDD: $\int_1^3 x \, dx = \frac{x^2}{2} \Big|_1^3 = \frac{3^2}{2} - \frac{1^2}{2} = 4$

$$\int_a^b f(x) \, dx = - \int_b^a f(x) \, dx$$

$$\int_0^\pi \sin x \, dx = -\cos x \Big|_0^\pi = -\cos \pi - (-\cos 0) = 1 + 1 = 2$$

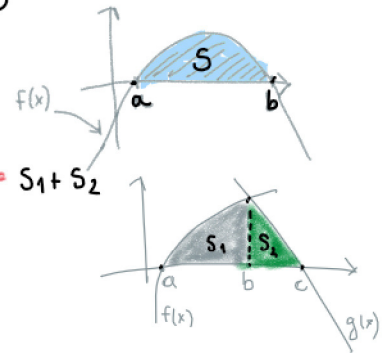
$$\int_1^4 \frac{x+\sqrt{x}+1}{x} \, dx = \int_1^4 \left(\frac{x}{x} + \frac{\sqrt{x}}{x} + \frac{1}{x} \right) dx = \int_1^4 \left(1 + x^{-\frac{1}{2}} + \frac{1}{x} \right) dx =$$

$$= \left(x + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \ln x \right) \Big|_1^4 = \left(4 + 2 + \ln 4 \right) - \left(1 + 2 + \ln 1 \right) = 3 + \ln 4$$

4. Določi ploščino odseka, ki ga omejujeta os x in parabola $y = x^2 - 5x + 4$.
5. Izračunaj ploščino lika, ki ga omejujejo parabola $xy = 6$ in premica $x + y = 7$.
6. Izračunaj ploščino lika med krivuljama $y = x^2$ in $y = 2 - x^2$.
7. Določi ploščino lika, ki ga krivulja $y = \frac{2x}{1+x^2}$ oklepa z osjo x , ničlo in maksimumom.
8. Izračunaj ploščino lika, ki ga omejujejo krivulje $y = \frac{\ln x}{x}$, os x , $x = 1$ in $x = e$.
9. Izračunaj ploščino lika, ki ga oklepata premici $y = x$ in $y = -\frac{1}{2}x + \frac{3}{2}$ z osjo x .
10. Izračunaj ploščino lika, ki ga funkciji $f(x) = -2x + 7$ in $g(x) = \sqrt{2x-1}$ oklepata z osjo x .
11. Izračunaj dolžino loka na krivulji $y = \sqrt{1-2x}$ na intervalu $[0, \frac{1}{2}]$.
12. Izračunaj prostornino vrtenine, ki nastane, ko zavrtimo za 360° okoli osi x , omejen z osjo x in krivuljo $y = 2x - x^2$ med presečiščema z osjo x .
13. Izračunaj prostornino vrtenine, ki nastane, ko zavrtimo za 360° okoli osi x , omejen z osjo x in krivuljo $xy = 1$ in pravokotnicama na os x v točkah $x_1 = 1$ in $x_2 = 10$.
14. Izračunaj prostornino vrtenine, ki nastane, ko zavrtimo za 360° okoli osi x , omejen s krivuljama $y = \frac{1}{4}x^2$ in $y = \frac{1}{4}x^3$.
15. Izračunaj koliko litrov vina lahko natočimo v parabolni sod višine 1,5 metra, največjim premerom 1,5 metra in najmanjšim premerom 1 metra. VIDEO

1.
$$\int_a^b f(x) \, dx = S$$

2.
$$\int_a^b f(x) \, dx + \int_b^d g(x) \, dx = S_1 + S_2$$



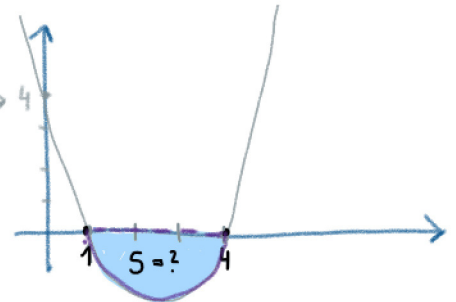
4. Izračunaj ploščino odseka, ki ga omejujeta os x in parabola $y = x^2 - 5x + 4$

1. skiciramo in poiščemo meje

$$y = x^2 - 5x + 4 = 0$$

$$(x-1)(x-4) = 0$$

$$x_1 = 1 \quad x_2 = 4$$



izračunamo ploščino:
$$\int_1^4 (x^2 - 5x + 4) \, dx = \frac{x^3}{3} - 5 \cdot \frac{x^2}{2} + 4 \cdot x \Big|_1^4 =$$

$$= \left(\frac{4^3}{3} - 5 \cdot \frac{4^2}{2} + 4 \cdot 4 \right) - \left(\frac{1^3}{3} - 5 \cdot \frac{1^2}{2} + 4 \cdot 1 \right) = -\frac{9}{2}$$

$$S = \left| -\frac{9}{2} \right| = \frac{9}{2}$$

5. Izračunaj ploščino lika, ki ga omejujeta krivulja $xy = 6$ in premica $x + y = 7$.

prekrižišče: $y = y$

$$\frac{6}{x} = -x + 7 \quad | \cdot x$$

$$6 = -x^2 + 7x$$

$$x^2 - 7x + 6 = 0$$

$$(x-6)(x-1) = 0$$

$$x_1 = 6 \quad x_2 = 1$$

meje

$$xy = 6$$

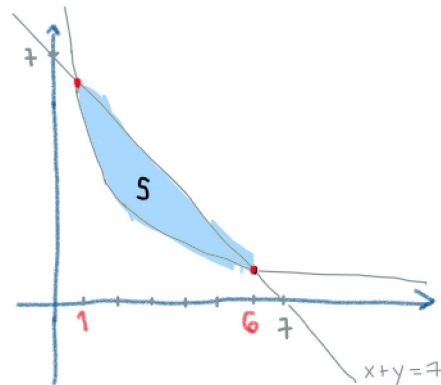
$$y = \frac{6}{x} = f(x)$$

x	y
1	6
2	3
6	1

$$x + y = 7$$

$$y = -x + 7 = g(x)$$

x	y
0	7
7	0



$$\int_1^6 (g(x) - f(x)) dx = \int_1^6 \left(-x + 7 - \frac{6}{x}\right) dx = \left[-\frac{x^2}{2} + 7x - 6 \ln x\right]_1^6 =$$

$$= \left(-\frac{6^2}{2} + 7 \cdot 6 - 6 \cdot \ln 6\right) - \left(-\frac{1}{2} + 7 - 6 \cdot \ln 1\right)$$

$$= \underline{\underline{\frac{35}{2} - 6 \cdot \ln 6}} > 0$$

Rešitev: $S = \frac{35}{2} - 6 \cdot \ln 6$

10. Izračunaj ploščino, lika ki ga funkciji $f(x) = -2x + 7$ in $g(x) = \sqrt{2x-1}$ oklepata z x osjo.

$$-2x + 7 = \sqrt{2x-1} \quad |^2$$

$$4x^2 - 28x + 49 = 2x - 1$$

$$4x^2 - 30x + 50 = 0$$

$$2x^2 - 15x + 25 = 0$$

$$(2x-5)(x-5) = 0$$

$$\downarrow \quad \downarrow$$

$$2x = 5 \quad x = 5$$

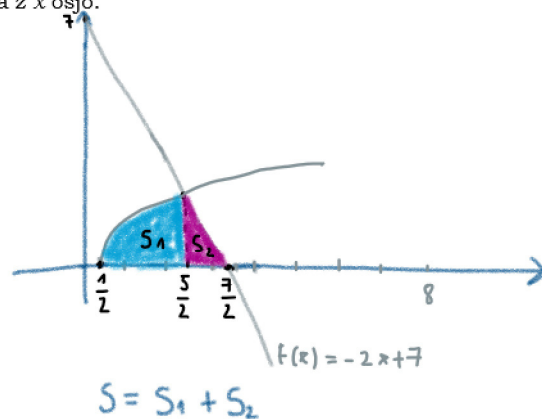
$$x = \frac{5}{2} \quad \checkmark$$

$$y = -2x + 7$$

x	y
0	7
$\frac{7}{2}$	0

$$y = \sqrt{2x-1}$$

x	y
1	1
5	3
$\frac{1}{2}$	0



$$\int_{\frac{1}{2}}^{\frac{5}{2}} (-2x + 7) dx + \int_{\frac{5}{2}}^{\frac{7}{2}} \sqrt{2x-1} dx = \left(-\frac{2x^2}{2} + 7x\right) \Big|_{\frac{1}{2}}^{\frac{5}{2}} + \frac{2}{3} \cdot \sqrt[3]{(2x-1)^2} \Big|_{\frac{5}{2}}^{\frac{7}{2}}$$

vstavimo meje

$$t = 2x - 1 \quad \sqrt{t} dt = \int t^{\frac{1}{2}} dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3} \cdot \sqrt[3]{t^2} = \frac{2}{3} \cdot \sqrt[3]{(2x-1)^2} \Big|_{\frac{5}{2}}^{\frac{7}{2}}$$

$$dt = 2 dx$$

$$dx = \frac{dt}{2}$$

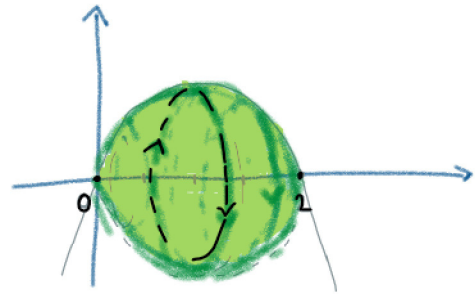
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12. Izračunaj ploščnino vrtenine, ki nastane, ko zavrtimo za 360° okoli osi x , omejene z osjo x in krivuljo $y = 2x - x^2$ med presečiščema z osjo x .

$$V = \pi \cdot \int_a^b f(x)^2 dx$$

$$y = 2x - x^2$$
$$y = x(2-x)$$

$x_1 = 0$ $x_2 = 2$



$$V = \pi \cdot \int_0^2 (2x - x^2)^2 dx = \pi \cdot \int_0^2 (4x^2 - 4x^3 + x^4) dx =$$
$$= \pi \cdot \left(\frac{4x^3}{3} - \frac{4x^4}{4} + \frac{x^5}{5} \right) \Big|_0^2 =$$
$$= \pi \cdot \left(\frac{4 \cdot 2^3}{3} - \frac{4 \cdot 2^4}{4} + \frac{2^5}{5} \right)$$
$$= \frac{16}{15} \pi$$

Več nalog, razlag in formul na [instrukcijeonline.com](https://www.instrukcijeonline.com)

