Nedoločeni integral: rešene naloge + formule

1. Integriraj naslednje funkcije.

(a) $\int (x^3 + x^2)\,dx$
(b) $\int 4\sin x\,dx$
(c) $\int (x^2 - 3x + 4)\,dx$
(d) $\int \sqrt{x}\,dx$
(e) $\int (x^2 - \frac{1}{x^2})\,dx$
(f) $\int \frac{x\sqrt{x} - x^2 - 5}{x^2\sqrt{x}}\,dx$
(g) $\int \frac{\sin(2x)}{2\sin x}\,dx$
(h) $\int e^x(1 - e^{-x}x^{-2})\,dx$
(i) $\int \frac{1-\sin^2 x}{\sin^2 x}\,dx$
(j) $\int \frac{\cos(2x)}{\cos x + \sin x}\,dx$
(k) $\int \sin^2 x\,dx + \int \cos^2 x\,dx$
(l) $\int e^{\ln x}\,dx$
(m) $\int \log_3 \sqrt[x]{3}\,dx$

uporabljeni so naslednji integrali:

$\int dx = x + C$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$\int \sin x\,dx = -\cos x + C$

$\int \cos x\,dx = \sin x + C$

$\int \frac{1}{x}\,dx = \ln x + C$

$\int e^x dx = e^x + C$

$\int 1\,dx = x + C$

$\ln e = 1$

$\int (x^3 + x^2)\,dx = \frac{x^4}{4} + \frac{x^3}{3} + C$

$\int 4\sin x\,dx = 4\cdot(-\cos x) = -4\cos x + C$

najprej v potenco: $\sqrt{x} = x^{\frac{1}{2}}$

$\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3}\cdot x^{\frac{3}{2}} = \frac{2}{3}\cdot\sqrt{x^3} + C$

$\sqrt[m]{x^n} = x^{\frac{n}{m}}$

$\int (x^2 - \frac{1}{x^2})\,dx = \frac{x^3}{3} + \frac{x^{-1}}{-1} = \frac{x^3}{3} - x^{-1} = \frac{x^3}{3} - \frac{1}{x} + C$ ($\frac{1}{x^2} = x^{-2}$)

$\frac{1}{x^n} = x^{-n}$

$\int \frac{x\sqrt{x} - x^2 - 5}{x^2\sqrt{x}}\,dx = \int \left(\frac{x\sqrt{x}}{x^2\sqrt{x}} - \frac{x^2}{x^2\sqrt{x}} - \frac{5}{x^2\sqrt{x}}\right)dx =$ na več posameznih ulomkov

$= \int \left(\frac{1}{x} - \frac{1}{\sqrt{x}} - \frac{5}{x^2\sqrt{x}}\right)dx = \ln x - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - 5\cdot\frac{x^{-\frac{3}{2}}}{-\frac{3}{2}}$ ($\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$)

$= \ln x - 2\sqrt{x} - \frac{10}{3}\cdot\frac{1}{\sqrt{x^3}} + C$

pomagaj si z dvojnim ulomkom

dvojni ulomek: $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a\cdot d}{b\cdot c}$

$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

$\frac{x^m}{x^n} = x^{m-n}$

$x^m \cdot x^n = x^{m+n}$

$x^2\cdot\sqrt{x} = x^2\cdot x^{\frac{1}{2}} = x^{\frac{5}{2}}$

$\int 5\cdot\frac{1}{x^2\sqrt{x}}\,dx = 5\cdot\int \frac{1}{x^2\sqrt{x}}\,dx = 5\cdot\int \frac{1}{x^{\frac{5}{2}}}\,dx = 5\cdot\frac{x^{-\frac{3}{2}}}{-\frac{3}{2}} = -\frac{10}{3}\cdot x^{-\frac{3}{2}}$

$\int \frac{\sin 2x}{2\sin x}\,dx = \int \frac{2\sin x\cdot\cos x}{2\sin x}\,dx = \int \cos x\,dx = \sin x + C$

- pomembne formule:

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\sin^2 x + \cos^2 x = 1$

$\int e^x(1 - e^{-x}x^{-2})\,dx = \int (e^x - x^{-2})\,dx = e^x - \frac{x^{-1}}{-1}$ (pomnožiš)

$= e^x + \frac{1}{x} + C$

$e^x\cdot e^{-x} = e^0 = 1$

$\int \frac{1 - \sin^2 x}{\sin^2 x}\,dx = \int \left(\frac{1}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x}\right)dx$

$= -\operatorname{ctg} x - x + C$

opomba! v nekaterih nalogah $1 - \sin^2 x$ zamenjamo s $\cos^2 x$, ampak v tem primeru ne gre rešit

$\ldots = \int \frac{\cos^2 x}{\sin^2 x}\,dx = \int \left(\frac{\cos x}{\sin x}\right)^2 dx = \int \operatorname{ctg}^2 x\,dx =$ // NE ZNAMO INTEGRIRATI

$\int \log_3 \sqrt[x]{3}\,dx = \int \log_3 (3^{\frac{1}{x}})\,dx$

$\log_a(a^x) = x$

$= \int \frac{1}{x}\,dx = \ln x + C$