Nedoločeni integral (uporaba nove spremenljivke) - rešene naloge

2. Z vpeljavo nove spremenljivke izračunaj nedoločene integrale.

(a) $\int (x^2+1)^7 2x\, dx$
(b) $\int (x+1)^5\, dx$
(c) $\int \sin(5x-2)\, dx$
(d) $\int \sqrt{10x-3}\, dx$
(e) $\int \sin^3 x \cos x\, dx$
(f) $\int x e^{-x^2}\, dx$
(g) $\int \frac{dx}{x+2}$
(h) $\int \frac{5dx}{\cos^2(3x)}\, dx$
(i) $\int \frac{x^2}{8-x^3}\, dx$
(j) $\int \frac{x\, dx}{(x^2+1)^2}$
(k) $\int \frac{e^x}{3e^x+2}\, dx$
(l) $\int (2+3\ln x)\frac{dx}{x}$
(m) $\int \frac{dx}{x\ln x}$
(n) $\int (x^2+5x-7)^{10}(2x+5)\, dx$
(o) $\int \frac{\sin(\ln x)}{2x}\, dx$
(p) $\int \frac{\sqrt{1+\ln x}}{2x}\, dx$

e, n ... podoben princip

samo to kar je v oklepaju

$$\int \underbrace{(x^2+1)}_{t}{}^7\, 2x\, \underbrace{dx}_{\frac{dt}{2x}} = \int t^7 \cdot \cancel{2x} \cdot \frac{dt}{\cancel{2x}} = \int t^7 dt = \frac{t^8}{8} + C = \boxed{\frac{(x^2+1)^8}{8} + C}$$

vedno se moramo v celoti znebiti spremenljivke x

$x^2+1=t$
$2x\,dx = dt$
$dx = \frac{dt}{2x}$

$$\int \underbrace{\sqrt{10x-3}}_{t}\, dx = \int \sqrt{t} \cdot \frac{dt}{10} = \frac{1}{10} \cdot \int \sqrt{t}\, dt = \frac{1}{10} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{10} \cdot \frac{2}{3} \cdot t^{\frac{3}{2}}$$

$$= \boxed{\frac{1}{15} \cdot \sqrt{t}^3 = \frac{1}{15} \cdot \sqrt{10x-3}^3 + C}$$

! brez korena vedno vzamemo

$10x-3=t$
$10\,dx = dt$
$dx = \frac{dt}{10}$

$$\int \underbrace{\sin^3 x}_{(\sin x)^3}\cos x\, dx = \int t^3 \cdot \cancel{\cos x} \cdot \frac{dt}{\cancel{\cos x}} = \frac{t^4}{4} + C = \boxed{\frac{\sin^4 x}{4} + C}$$

$(\underbrace{\sin x}_{t})^3$ brez eksponenta vzamemo

$\sin x = t$
$\cos x\, dx = dt$
$dx = \frac{dt}{\cos x}$

$$\int \frac{5\, dx}{\cos^2(3x)} = 5 \cdot \int \frac{1}{\cos^2 t} \cdot \frac{dt}{3} = \frac{5}{3} \cdot tg(t) + C = \boxed{\frac{5}{3} \cdot tg(3x) + C}$$

↳ isto kot $\int \frac{5}{\cos^2(3x)}\, dx$

$3x = t$
$3\,dx = dt$
$dx = \frac{dt}{3}$

$$\int \frac{x^2}{\underbrace{8-x^3}_{t}}\, dx = \int \frac{\cancel{x^2}}{t} \cdot \frac{dt}{-3\cancel{x^2}} = -\frac{1}{3} \int \frac{dt}{t} = -\frac{1}{3} \ln|t|$$

$$= \boxed{-\frac{1}{3} \cdot \ln|8-x^3| + C}$$

$8-x^3=t$
$-3x^2\, dx = dt$
$dx = \frac{dt}{-3x^2}$

$$\int \frac{e^x}{\underbrace{3e^x+2}_{t}}\, dx = \int \frac{\cancel{e^x}}{t} \cdot \frac{dt}{3\cancel{e^x}} = \frac{1}{3}\int \frac{dt}{t}$$

$$= \frac{1}{3} \cdot \ln|t| + C = \boxed{\frac{1}{3} \cdot \ln(3e^x+2) + C}$$

$3e^x+2=t$
$3e^x\, dx = dt$
$dx = \frac{dt}{3e^x}$

$$\int (2+3\ln)\cdot\frac{1}{x}\cdot dx \qquad\qquad \int\frac{dx}{x}=\int\frac{1}{x}\,dx=\ln x$$

$$dx \to \frac{x\cdot dt}{3}$$

$$\int \underbrace{(2+3\ln x)}_{t}\frac{dx}{x} = \int t\cdot\frac{\frac{x\cdot dt}{3}}{x} = \int t\cdot\frac{\cancel{x}\cdot dt}{3\cancel{x}} = \frac{1}{3}\int t\cdot dt$$

$$= \frac{1}{3}\cdot\frac{t^2}{2}+C = \boxed{\frac{1}{3}\cdot\frac{(2+3\ln x)^2}{2}+C}$$

$$2+3\ln x = t$$

$$\frac{3}{x}\,dx = dt$$

$$dx = \frac{dt}{\frac{3}{x}} = \frac{x\cdot dt}{3}$$

$$\int\frac{\sin(\ln x)}{2x}\,dx = \int\frac{\sin t}{2\cancel{x}}\cdot\cancel{x}\cdot dt = \frac{1}{2}\cdot(-\cos t)+C =$$

$$= \boxed{-\frac{1}{2}\cdot\cos(\ln x)+C}$$

$$\ln x = t$$

$$\frac{1}{x}\,dx = dt$$

$$dx = x\cdot dt$$