UPORABA DOLOČENEGA INTEGRALA (nekaj nalog)

1. Izračunaj ploščino lika, ki ga premica $2x + y - 6 = 0$ oklepa:

a) Z osjo x na intervalu [1,4]

b) Z obema koordinatnima osema

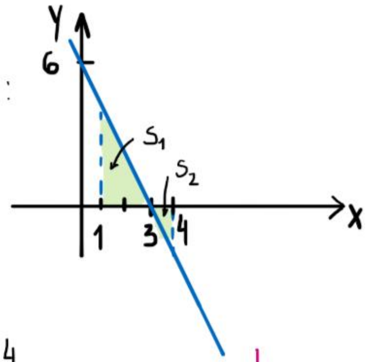
enačba premice:

$$2x + y - 6 = 0$$

$$y = -2x + 6$$

v eksplicitno obliko enačbe premice

skica:



a)

$$S = S_1 + S_2 = \int_1^3 (-2x+6)\,dx + \left| \int_3^4 (-2x+6)\,dx \right|$$

$$= \left(-\frac{2x^2}{2} + 6x\right)\Big|_1^3 + \left| \left(-\frac{2x^2}{2} + 6x\right)\Big|_3^4 \right|$$

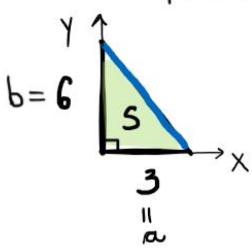
$$= -9 + 18 - (-1 + 6) + \left| -16 + 24 - (-9 + 18) \right|$$

$$= 4 + |-1| = 4 + 1 = 5$$

b)

1.način: $$\int_0^3 (-2x+6)\,dx = \left(\frac{-2x^2}{2} + 6x\right)\Big|_0^3 = -9 + 18 - 0 = 9$$

2.način: ploščina pravokotnega trikotnika



$$S = \frac{a \cdot b}{2} = \frac{6 \cdot 3}{2} = 9$$

17. Izračunaj ploščino lika, ki ga omejujejo krivulja $y = x^{-1}\ln x$, os x in premici x = 1 in x = e.

$$\boxed{y = x^{-1}\ln x = \frac{\ln x}{x}}$$

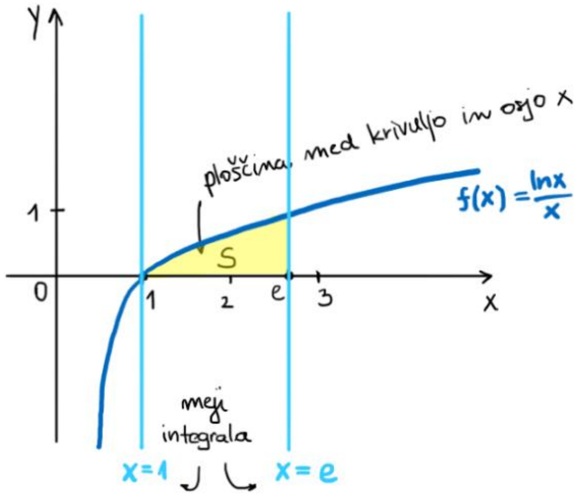
ničle: $\ln x = 0$

$x = 1$

Definicijsko območje: $x > 0$

$\boxed{\ln x \,;\, x > 0}$ $\quad x \neq 0$

x	y
1	0
2	$\frac{\ln 2}{2}$
$\frac{1}{2}$	$2\ln\left(\frac{1}{2}\right)$



Ploščina: $\displaystyle\int_1^e \frac{\ln x}{x}\,dx = \int_{g(1)}^{g(e)} \frac{t}{\cancel{x}}\cdot \cancel{x}\,dt = \frac{t^2}{2}\Big|_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$

nova spremenljivka

$t = \ln x = g(x)$

$dt = \frac{1}{x}\,dx$

$dx = x \cdot dt$

$g(1) = \ln 1 = 0$

$g(e) = \ln e = 1$

18. Izračunaj ploščino lika, ki ga omejujejo y = tgx, y = ctgx in os x v intervalu $\left[0, \frac{\pi}{2}\right]$.

$\boxed{y = \operatorname{tg} x}$ $\quad \boxed{y = \operatorname{ctg} x}$ $\quad$ na intervalu $\left[0, \frac{\pi}{2}\right]$

$y = \frac{\sin x}{\cos x}$

ničle: $\sin x = 0$

$x = 0 + k\pi$

poli: $\cos x = 0$

$x = \frac{\pi}{2} + k\pi$

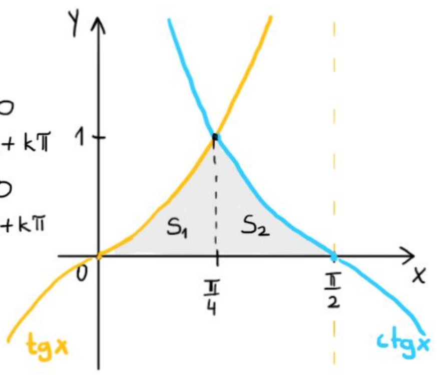
$y = \frac{\cos x}{\sin x}$

ničle: $\cos x = 0$

$x = \frac{\pi}{2} + k\pi$

poli: $\sin x = 0$

$x = 0 + k\pi$



Ploščina: $S = S_1 + S_2$

$$= \int_0^{\frac{\pi}{4}} \operatorname{tg} x\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \operatorname{ctg} x\,dx$$

$\cos x = t$

$-\sin x\,dx = dt$

$dx = \frac{dt}{-\sin x}$

$$= \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x}\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x}\,dx = \int_{\cos 0}^{\cos\frac{\pi}{4}} \frac{\cancel{\sin x}}{t}\cdot\frac{dt}{-\cancel{\sin x}} + \int_{\sin\frac{\pi}{4}}^{\sin\frac{\pi}{2}} \frac{\cancel{\cos x}}{t}\cdot\frac{dt}{\cancel{\cos x}}$$

$\sin x = t$

$\cos x\,dx = dt$

$dx = \frac{dt}{\cos x}$

$$= -\int \frac{dt}{t} + \int \frac{dt}{t} = -\ln t\Big|_1^{\frac{\sqrt{2}}{2}} + \ln t\Big|_{\frac{\sqrt{2}}{2}}^1 = -\left(\ln\frac{\sqrt{2}}{2} - \ln 1\right) + \ln 1 - \ln\frac{\sqrt{2}}{2} = \ln 2$$

UPORABA ODVODA (nekaj nalog)

52. Določi intervale naraščanja in padanja funkcije:

a) $f(x) = 2x^2 + 12x + 17$

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Definicijsko območje: $D_f = \mathbb{R}$

Odvod funkcije: $f'(x) = 2 \cdot 2x + 12 = 4x + 12$

Rešimo neenačbo: $f'(x) > 0$ → Rešitve te neenačbe je interval naraščanja

$4x + 12 > 0$

$4x > -12 \quad | : 4$

$x > -\frac{12}{4}$

$x > -3$

$\underbrace{x \in (-3, \infty)}_{\text{naraščanje}}$

Funkcija narašča na intervalu $(-3, \infty)$

$f'(x) < 0$

$\underbrace{x \in (-\infty, 3)}_{\text{padanje}}$

Funkcija pada na intervalu $(-\infty, -3)$

54. Določi stacionarne točke funkcije:

a) $\boldsymbol{f(x) = x^3 - 12x + 7}$

$f(x) = x^3 - 12x + 7$

stacionarne točke
$f'(x) = 0$

Odvod funkcije: $f'(x) = 3x^2 - 12 = 0$

odvod enačimo z nič

$3x^2 - 12 = 0 \quad |:3$

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$x_1 = -2 \qquad x_2 = 2$

$f(-2) = -8 + 24 + 7$

$f(2) = 8 - 24 + 7$

Stacionarne točke: $T_1(-2, 23)$

$T_2(2, -9)$

55. Določi lokalne ekstreme funkcije

a) $f(x) = 3x^4 + 20x^3 + 25$

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Odvod funkcije: $f'(x) = 3 \cdot 4x^3 + 20 \cdot 3x^2 = 12x^3 + 60x^2$

Ničle odvoda: $f'(x) = 0$

$$12x^3 + 60x^2 = 0 \quad /:12$$
$$x^3 + 5x^2 = 0$$
$$x^2(x+5) = 0$$
$$x_{1,2} = 0 \qquad x_3 = -5$$

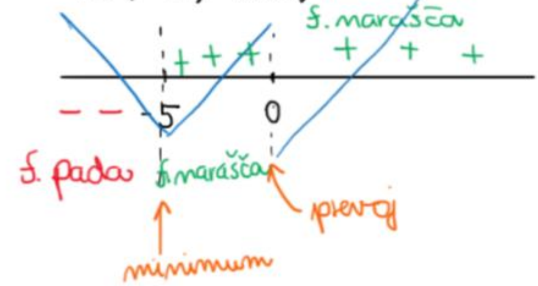
$f(0) = 25$

$f(-5) = 3 \cdot (-5)^4 + 20 \cdot (-5)^3 + 25 = -600$

Stacionarne točke: $T_1(0, 25)$ prevoj

$T_2(-5, -600)$ minimum

Predznaki odvoda:



Funkcija ima v točki $x = -5$ lokalni minimum -600.